

$$\sum_{i=0}^k 2^i = 2^{k+1} - 1 \geq N$$

$$k \geq \log_2(N+1) - 1$$

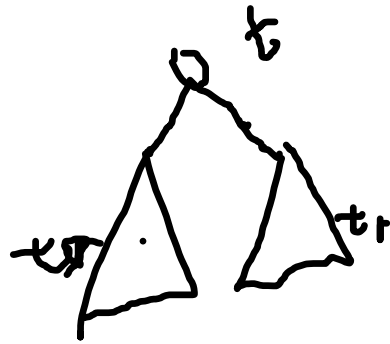
highest = 10 of 10

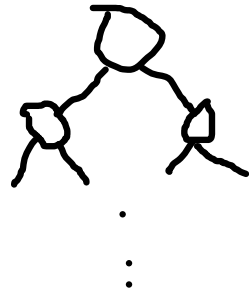
0 1 2

0 1 2

0

{1, 2, 3}

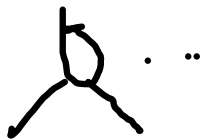




$k$   
 $0$   
 $1$   
 $\vdots$   
 $\vdots$

$N$   
 $1$   
 $2$   
 $\vdots$   
 $\vdots$

$$N = 2^{(k+1)} - 1$$



$k$

$2^k$

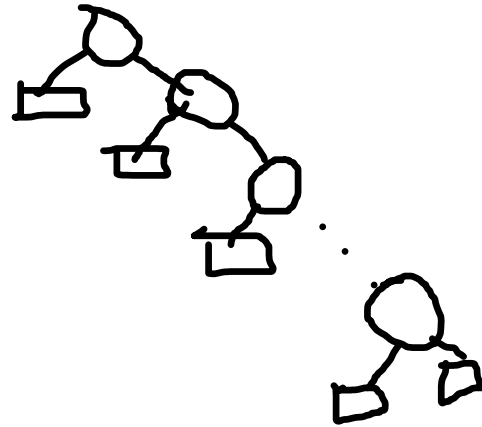
$$I(t) = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 4 + 4 \cdot 8 + \dots + (k+1) \cdot 2^k$$

$$2 \cdot I(t) = 1 \cdot 2 + 2 \cdot 4 + 3 \cdot 8 + \dots + k \cdot 2^k + (k+1) \cdot 2^{(k+1)}$$

$$- I(t) = 1 + 2 + 4 + 8 + \dots + 2^k - (k+1) \cdot 2^{(k+1)}$$

$$= 2^{(k+1)} - 1 - \dots$$

$$I(t) = (k+1) \cdot 2^{(k+1)} - 2^{(k+1)} + 1 = k \cdot (N+1) + 1 = \log_2(N+1) \cdot (N+1) + 1 = O(N \log N)$$



$$I(t) = 1 + 2 + 3 + \dots + N-1 = N(N-1)/2 = O(n^2)$$